

Endomorphisms in Persistent Homology and the Conley Index Computation via Quiver Representations

Michał Palczewski

*Faculty of Applied Physics and Mathematics & Doctoral School,
Gdańsk University of Technology, Gdańsk, Poland*
`michal.palczewski@pg.edu.pl`

We introduce a method for the computation of the homomorphism induced in persistent homology by a continuous map, building on ideas from previous approaches [1, 2, 4]. The method combines a continuous function $f: X \rightarrow Y$ with the inclusion map $\iota: X \rightarrow Y$, allowing for the computation of the Conley index map, as discussed in [3]. Given topological spaces X and Y , along with sampled data $f|_S$ representing the restriction of f to a finite subset $S \subset X$, we propose a framework based on quiver representations for the computation of an endomorphism in persistent homology. We introduce the notion of approximate index pairs based on the sampled map, and we apply our framework to make an attempt to compute a persistence version of the Conley index.

References

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