

CMVF - C++ library for multivectors fields

Jakub Leśkiewicz

*Faculty of Mathematics and Computer Science
Jagiellonian University, Kraków, Poland
jakub.leskiewicz@doctoral.uj.edu.pl*

The multivector fields theory is a dynamically developing research area. It is a convenient way to represent multivalued, discrete dynamical systems in finite topological spaces. The main idea was presented by Prof. Marian Mrozek in [4]. As all objects in the theory are purely combinatorial, they can be easily represented in software.

During the Combinatorial Topological Dynamics conference, I want to present a C++ library that I have developed over the last five years. It is a large template library designed to implement the main functionalities of multivector field theory.

It enables the creation of multivector fields on Lefschetz complexes (with or without geometric realization), calculates the homological Conley index of multivectors, and finds Morse decompositions and Poincaré sections.

Moreover, it implements all the algorithms from [1], so it can be easily used in computer-assisted proofs in dynamics, in the spirit of the approach presented in [2]. The two-dimensional case has recently been finished and optimized, while the three-dimensional case is still a work in progress.

In recent months, it has also been supplemented with additional algorithms related to the analysis of sampled dynamics, described in a yet unpublished paper [3]. Thanks to the software's support for Gaussian processes, it enables reliable analysis of sampled flows.

As Forman Vector Fields are a special case of Multivector Fields, the software also provides the opportunity to perform topological data analysis, calculate Depth Posets, and make topological simplifications in accordance with [8] and [7].

The C++ library comes with software that allows for convenient visualization of two- and three-dimensional multivector fields.

References

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