From Data to Combinatorial Dynamical System Through Optimization

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Combinatorial dynamical systems in the sense of Forman and combinatorial multivector fields can be useful to study data from dynamical systems. One problem is that is not well suited to approximate any dynamical systems from data. In this poster, we show different three different linear optimization problems with binary variables. We have the following data. We have a simplicial complex K embedded in $\mathbb{R}^n$, and for each simplex, we have a value in $\mathbb{R}$. The first optimization problem goes as follows. The variables are a pair of simplices with binary values. If the variable has the value one, then the simplices are in the same matching. We assign a cost to each variable by using the data. If the cost is low, then the simplices should be in the same matching. We add a parameter α to decide if a simplex is critical or not. We minimize the cost, and the optimal solution induces a discrete vector field. We can show that it can be solved in polynomial times. We generalize the first problem to the multivector field case. We had a new parameter β to change the size of multivector. There exists values α and β such that the optimal solution induces a combinatorial multivector field. A major problem is the solving time is NP-hard. Therefore, we simplify it to obtain the third one. We remove all parameters, and each combinatorial multivector contains a single toplex. We still obtain nice results. We conjecture that it can be solved in polynomial times. We apply our different methods on classical dynamical systems such as the Lotka-Volterra system and the Lorenz attractor. We are able to extract the main features.

References

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