

The height of the depth poset

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The Depth Poset is an additional poset structure over the persistence diagram. In analogy with perfect Morse functions in Forman-Morse theory, we are interested in flat depth posets, that is ones with no non-trivial relations. As in the former case, a simplicial complex is collapsible if and only if it has the homology of the point and admits a filtration inducing a flat depth poset. Generalizing this concept, we introduce the height of the depth poset as the length of its maximal chain minus one. This height measures how much the associated filter "fails to be perfect". We show examples of topological spaces, which, with any triangulation and any filter, cannot admit a flat depth poset – the Dunce hat and Poincaré homology sphere. In order to prove this result we introduce a canonical form of the filter, that is a filter such that shallow pairs (minimal elements of the depth poset) are contiguous.

This is a work in progress in collaboration with Manuel Soriano-Trigueros.

References

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